

Quasars: a supermassive rotating toroidal black hole interpretation.

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ABSTRACT

A supermassive rotating toroidal black hole (TBH) is proposed as the fundamental structure of quasars and other jet-producing active galactic nuclei. Rotating protogalaxies gather matter from the central gaseous region leading to the birth of massive toroidal stars whose internal nuclear reactions proceed very rapidly. Once the nuclear fuel is spent, gravitational collapse produces a slender ring-shaped TBH remnant. Transitory electron and neutron degeneracy stabilised collapse phases, though possible, are unlikely due to the large masses involved thus these events are typically the first supernovae of the host galaxies. Given time, the TBH mass increases through continued accretion by several orders of magnitude, the event horizon swells whilst the central aperture shrinks. The difference in angular velocities between the accreting matter and the TBH induces a magnetic field that is strongest in the region of the central aperture and innermost ergoregion. Due to the presence of negative energy states when such a gravitational vortex is immersed in an electromagnetic field, circumstances are near ideal for energy extraction via non-thermal radiation including the Penrose process and superradiant scattering. This establishes a self-sustaining mechanism whereby the transport of angular momentum away from the quasar by relativistic bi-directional jets reinforces both the modulating magnetic field and the TBH/accretion disk angular velocity differential. Continued mass-capture by the TBH results in contraction of the central aperture until the TBH topology transitions to being spheroidal, extinguishing quasar behaviour. Similar mechanisms may be operating in microquasars, supernovae and sources of repeating gamma ray bursts when neutron density or black hole tori arise. Long-term TBH stability seems to require either a negative cosmological constant, a non-stationary spacetime due to the presence of accreting matter or the intervention of quantum gravitational effects.

Key words: quasars: general – black hole physics – galaxies: active – stars: neutron – gamma ray: bursts – supernovae: general.

1 INTRODUCTION

The commonly cited method of producing bi-directional jets as observed in quasar objects is the mechanism described by Blandford & Znajek (1978), whereby magnetic field lines thread the poles of a rotating BH as they descend towards the event horizon. Rotational energy may be extracted from the hole by this technique which is ejected in the form of radiation and matter travelling at high velocity along the hole's spin axis. Of recent note are critical assessments by Ghosh & Abramowicz (1997) and Livio, Ogilvie and Pringle (1999) which suggest that the role of the Blandford-Znajek mechanism has been generally overestimated and inadequately accounts for the larger double radio lobe structures. By virtue of numerical simulations we now know that the observed gamma ray energy release along quasar jets is four orders of magnitude more energetic than can be explained solely by the Blandford-Znajek mechanism. Issues that are difficult to reconcile with this model are the variability of jet dispersion angles, the finite quasar lifetime and the multiplicity of red-shifts in the very metallic absorption spectra. Numerous other models have been proposed, none of which have successfully catered for all features simultaneously.

Speculation concerning the fundamental processes governing quasars invariably involves discussion of compact massive central bodies, the consensus being that these are rotating spheroidal BHs of mass $10^6 - 10^9 M_\odot$. Profiles of stellar orbital velocities within AGN haloes lend weight to the premise that a massive object resides at the galactic nucleus. Inactive galactic nuclei have also yielded similar velocity profiles and these are difficult to reconcile with current AGN models. Very little progress seems to have been made over

past decades to bring us nearer a full explanation for these exceedingly energetic phenomena. Exploring new physics may be the only avenue available if we are to arrive at a consistent description of quasars and AGN in general. Yet, the predictions of alternative physical theories must not contradict the experimental data hitherto obtained. The purpose of this discussion is to advocate a new mechanism, illustrate its operation and describe how, if a negatively valued cosmological constant term is permitted, a satisfactory explanation for these phenomena may be attained.

In this text, the similarities between a rotating toroidal black hole (TBH) at the galactic centre accreting matter from its surroundings will be compared with observational evidence from quasars, Seyferts, BL Lacertae and blazars, which have long been suspected to be manifestations of the same underlying astrophysical objects. Attention will be paid to the formation of such a TBH, its long-term stability in our universe, jet generation processes and its evolution with time. We shall examine why it might be that quasars were so widespread in the early universe that almost every galaxy may have played host to one. Numerical illustrations along with a diagram encapsulating the life-cycles of toroidal BHs, in close agreement with those of quasars are finally presented.

2 ROTATING TOROIDAL BLACK HOLES AND THEIR FORMATION

It is proposed that the central component of the quasar mechanism is a rapidly rotating black hole with a toroidal event horizon. First, the issue of how such an object may come into being shall be addressed. The majority of observable galaxies are rotating ones that have stabilised over time in such a way that the orbiting stars are highly concentrated around the plane of galactic rotation. We know from direct observations by the COBE satellite that matter was very evenly distributed throughout the cosmos in early eras. On the scale of inter-galactic distances, matter would then have collapsed slowly under the action of gravity, and protogalaxies composed of low-density hydrogen and helium gas would have emerged. Because most observable galaxies are known to rotate somewhat, we would anticipate these protogalaxies to generally have possessed some angular momentum.

As gravity shapes these rotating protogalaxies the gas is drawn towards the plane of rotation. The gas particles have random velocities and collisions are relatively infrequent due to the low volumetric mass density. Those with small velocities tend to be drawn towards the centre of the galaxy due to net gravitational attraction. However, these particles gain kinetic energy as they proceed towards the centre, which means that very few will be able to occupy orbits confined to the centremost regions of the galaxy. Instead they tend to accumulate in elliptical orbits of larger radii and less matter is actually present at the exact centre of rotation, see Fig.1 curve labelled $t = 1$. For a given instant in time, as we progress along the galactic plane from the precise centre of the galaxy we find that the molecular density initially increases to a maximum value and thereafter tapers off. As time progresses, the gas distribution becomes more pronounced and collisions between molecules more frequent. The evolution of the gas at the galactic centre according to the present model is qualitatively depicted by the series of gas density curves in Fig.1. The curve labelled $t = 4$ represents what may be identified as a toroidal gas cloud. This toroidal gas cloud continues to condense as more gas molecules amass until the density and pressure resemble that of a conventional star, at which time nuclear fusion begins. The toroidal stars of relevance to this model, see Fig.2 (a), must satisfy several conditions: a brief lifetime, be sufficiently massive and dimensionally large so that enough mass remains following a supernova (SN) implosion to form a TBH.

Drawing on comparisons with what is known about neutron stars and the limits on their size, we now estimate the upper limit for the minor radius R_2 of a neutron ring above which collapse to a TBH will result. A few simplifying assumptions are necessary. First, the density of neutron star material is assumed to remain constant and independent of pressure (incompressible fluid). Hydrostatic equilibrium is reached whereby the pressure of the fluid counteracts gravitational compression at all locations. Newtonian approximations will be used to derive the surface gravity. The torus is assumed to have a major radius much larger than the minor radius, $R_1 \gg R_2$, so that an infinitely long cylinder approximation is valid. Both neutron stars are assumed to have negligible rotation. The gravitational field within a sphere of constant density tails off linearly from the surface to the centre, even according to general relativity. We begin by confirming that gravity within an infinitely long solid cylinder of constant mass density is linearly related to the radial distance from the central axis using a Newtonian analysis.

We know that for spherical masses, once we are external to the object, the entire mass may be assumed

to be located at the centre of mass. Similarly, a solid cylinder can be regarded as a line mass with some constant mass per unit length provided we are outside the cylinder. Let us assign a radius to the cylinder of b , a longitudinal coordinate x and angular coordinate ϑ . We wish to calculate the gravitational field strength at some radius a with $x = 0$ and $\vartheta = 0$ given that $a < b$. We are therefore external to a cylinder of radius a and internal to a cylindrical shell of radial thickness $b - a$. Because Newtonian gravity obeys the principle of superposition we reduce the problem to showing that the gravitational field vanishes at all points located within an infinitesimally thin and infinitely long cylindrical shell with constant mass per unit area σ . Then, using the result which is later derived for the surface gravity of a cylinder, equation (5), we see that gravity inside an infinite cylinder is directly proportional to the radial coordinate.

Consider the field within a thin shell at some radius, a . Integrating the radially directed gravitational field contributions of elemental shell masses we find that the expression is: -

$$g(a) = 2G\sigma \int_0^\pi \int_{-\infty}^{+\infty} \frac{b(a - b \cos \vartheta)}{(a^2 + b^2 - 2ab \cos \vartheta + x^2)^{3/2}} dx d\vartheta \quad (1)$$

Integrating with respect to x gives: -

$$g(a) = 2G\sigma \int_0^\pi \left[\frac{x}{\sqrt{x^2 + a^2 - 2ab \cos \vartheta + b^2}} \right]_{-\infty}^{+\infty} \frac{ab - b^2 \cos \vartheta}{a^2 - 2ab \cos \vartheta + b^2} d\vartheta = 4G\sigma \int_0^\pi \frac{ab - b^2 \cos \vartheta}{a^2 - 2ab \cos \vartheta + b^2} d\vartheta \quad (2)$$

Splitting the integral in two and integrating with respect to ϑ we get: -

$$g(a) = 4\pi G\sigma ab \left(\frac{1}{|b^2 - a^2|} - \frac{1}{b^2 - a^2} \right) \quad (3)$$

From which we see that providing $|a| < |b|$ then $g(a)$ is zero. It is therefore possible to ignore the gravitational contribution of cylindrical shells with radii larger than ours and consider only the cylinder with radius a which can be reduced to a line mass since we lie on its surface. When a line mass is considered, the expression for surface gravity given in equation (5) shows that gravity within an infinitely long cylinder is directly proportional to the radial coordinate.

We shall assign a constant mass density ρ to both the neutron stars. The radius of the neutron sphere is R_S and the minor radius of the neutron ring R_R . Surface gravity for the sphere g_S and for the ring g_R are assigned. We can immediately calculate the surface gravity for the sphere: -

$$g_S = \frac{GM_{\text{sphere}}}{R_S^2} = \frac{4\pi\rho GR_S}{3} \quad (4)$$

The surface gravity of the infinitely long cylinder (x -coordinate) is obtained by integrating the gravitational contribution of infinitesimal line mass elements to a point at a distance R_R from the line. The mass of the line is $\pi\rho R_R^2$ per unit length. The contribution of the infinitesimal line mass elements to the gravitational acceleration at the surface need only be considered in the direction perpendicular to the infinite line mass. Hence, we have the following expression: -

$$g_R = \pi\rho GR_R^3 \int_{-\infty}^{+\infty} \frac{dx}{(x^2 + R_R^2)^{3/2}} = 2\pi\rho GR_R \quad (5)$$

By calculating the pressure at the centre of the sphere (P_{SC}) and ring (P_{RC}), then equating the two values, it will be possible to compare the limiting radii at which further gravitational collapse takes place. Assuming that the surface pressures are zero, we need to integrate over infinitesimally thin (spherical or cylindrical) shells of matter. The pressure difference between the inner and outer surface of a shell is given by the weight of the shell divided by the area of the inner shell surface. Noting that the weight of the shell depends on the local value of gravity, which is constant throughout the shell and a linear function of radius from zero at the centre to g_S or g_R at the surface, we can write: -

For the sphere,

$$P_{SC} = \frac{\rho g_S}{R_S} \int_0^{R_S} r dr = \frac{\rho g_S R_S}{2} \quad (6)$$

For the ring,

$$P_{RC} = \frac{\rho g_R}{R_R} \int_0^{R_R} r dr = \frac{\rho g_R R_R}{2} \quad (7)$$

Equating P_{SC} and P_{RC} we then get a value for the maximum neutron ring minor radius in terms of the maximum neutron star radius. Note that this result is independent of the density of neutron star matter and that the reliability of the result is improved by the balancing of the Newtonian approximations: -

$$R_{ring_{max}} = \sqrt{\frac{2}{3}} \times R_{sphere_{max}} \approx 8.5 km \quad (8)$$

As we would expect, the minor radius of a neutron ring must be smaller than the maximum spherical case. If general relativity were to be used then the pressure gradient for a spherical star would be given by the standard equation describing hydrostatic equilibrium: -

$$\frac{dP}{dr} = -G \frac{(\rho + P/c^2)[m(r) + 4\pi r^3 P/c^2]}{r[r - 2Gm(r)/c^2]} \quad (9)$$

Here P is the pressure at some radius r and $m(r)$ is the mass enclosed by the 2-sphere defined by r which need not necessarily be the formula for the mass of a sphere radius r if an equation of state is available. We note only that the introduction of general relativity makes it more difficult for neutron degeneracy to withstand gravity, but the estimate of equation (8) is adequate for our present discussion.

Assuming that the pre-collapse seed star is composed of the same materials as our sun and is incompressible, its density will be $\sim 1400 kg m^{-3}$. We shall limit the discussion to those toroidal stars that are capable of gravitational implosion directly to a TBH rather than those with intermediate white dwarf or neutron density phases. For the purposes of approximation, the surface areas of extremal Kerr BH event horizons are equated with the surface areas of TBHs with equal mass, alternatively this may be viewed as equating the entropy of the BHs. The TBH is assumed to have an angular momentum equal to the extremal Kerr BH of equal mass. In addition, the TBH geometry will be taken to be that of an Euclidean torus parametrized by the major and minor radii R_1 and R_2 respectively. This crude model permits us to prepare some order of magnitude estimates.

An extremally rotating Kerr hole has $r_+ = m$ so its area is $A = 4\pi r_+^2 = 4\pi m^2$. The surface area of an Euclidean torus is $A = 4\pi^2 R_1 R_2$ so, to a good approximation, we can relate the rotating TBH mass to the TBH area by equating these two expressions for area and, after restoring the natural constants ($r_+ = Gm/c^2$), we get: -

$$M_{TBH} \approx \frac{c^2}{G} \sqrt{\pi R_1 R_2} \quad (10)$$

We now consider a (low density) toroidal star (TS) of Euclidean geometry whose major radius is R_1 as before, but with a minor radius R_3 . Evidently $R_3 > R_2$ otherwise the TS is a TBH and $R_3 < R_1$ ensures the star is toroidal. The TS undergoes gravitational collapse once its nuclear fuel is exhausted and the resulting TBH is assumed to have the same major radius R_1 as the TS. Since the volume of the TS is $V = 2\pi R_1 R_3^2$, and the TS is composed of constant density material ($\rho = 1400 kg m^{-3}$) then the mass of the toroidal star will be $M = 2\pi \rho R_1 R_3^2$. Following a SN implosion of the star, typically most of the mass will have been ejected. We assign the parameter η to represent the fraction of the original TS mass remaining in the TBH after the SN. We equate the remaining mass with the mass of the resultant TBH and obtain:

$$\frac{c^2}{G} \sqrt{\frac{R_2}{4\pi R_1}} \approx \rho \eta R_3^2 \quad (11)$$

We have already determined the maximum minor radius of a neutron ring in equation (8) so: -

$$R_1 > R_2 > \sim 8.5 km \quad (12)$$

Taking the limit as $R_3 \rightarrow R_1$ with $R_3 < R_1$ in equation (11) and setting $\eta = 0.1$ (90% mass ejection) gives a limit for TBH formation: -

$$\frac{R_1^5}{R_2} > \frac{c^4}{4\pi\rho^2\eta^2G^2} \approx 7.4 \times 10^{48} m^4 \quad (13)$$

Allowing $R_2 \rightarrow 8.5 km$ with the previous condition gives a lower bound for R_1 : -

$$R_1 > \sim 36 \times 10^9 m \quad (14)$$

If the TS grows too large and too massive, then it will become a TBH without an implosion or electron/neutron degeneracy supported phases. So we insist that the area of the TS be larger than the area of a TBH of the same mass and this leads to $R_3 > R_2$ since R_1 is common to both. Then, using the expression for R_3 and taking the limit as $R_3 \rightarrow R_2$ with $R_3 > R_2$ in equation (11) we find that: -

$$R_1 R_2^3 < \frac{c^4}{4\pi\rho^2\eta^2G^2} \approx 7.4 \times 10^{48} m^4 \quad (15)$$

The SN is assumed to eject 90% of the original star's mass during the implosion (this assumption is the least reliable and easily dominates the combined errors of the remaining assumptions). In special cases where R_2 approaches R_1 and the inequalities hold then almost no mass is ejected because the star does not collapse much before the event horizon engulfs it. In general though, the angular velocity of the resulting BH will match that of the seed star so we would expect less mass ejection than in more familiar SN events wherein a star collapses to form a spheroidal BH with very high angular velocity. For a given R_2/R_1 ratio, the permissible range of toroidal star masses which can gravitationally collapse to form a TBH range is typically quite broad (Fig.9), we shall return to this issue later.

These massive toroidal stars would have rapidly exhausted their nuclear fuel. The end result would be a supernova-like implosion, most likely the first SN event of its host galaxy, presumably localised to one portion of the ring initially, Fig.2 (b). Because the implosion cannot propagate faster than the speed of light, it could take several hours or days for the implosion to propagate around the torus in both directions, Fig.2 (c), until the implosion fronts meet at the opposite end of the torus. Similarly, the implosion causes a thin, string-like event horizon to propagate around the torus which finally encounters itself and seals to provide a stable TBH, Fig.2 (d). The illustrations of Fig.2 do not include the ejected matter and are not based upon precise physical calculations, they are intended only to show how the gravitational implosion is likely to progress around the ring.

The mass of the toroidal star is such that if as much as 90% of its mass is ejected during the SN implosion, there will still remain enough mass to construct what must inevitably become a BH rather than a neutron star remnant. Suppose that much more of the mass is ejected during the SN, perhaps 99%, then what may remain could conceivably be a toroidal white dwarf or toroidal neutron star. In either case, the galaxy is so young that this central region will be rapidly gathering matter from its neighbourhood so that a transition to toroidal neutron star and thereafter TBH will swiftly follow. Indeed, these transitions will be somewhat less violent than the direct stellar/BH implosion and more likely to preserve the integrity of the resulting event horizon as opposed to a break-up into individual co-orbiting spheroidal BHs. Although a toroidal neutron star could form in this way, its central aperture would necessarily be too large to support strong quasar-like behaviour and the brief lifespan it would experience before collapse to TBH (due to the high accretion rate) precludes such objects as serious quasar candidates. Smith and Mann (1997) have recently investigated gravitational collapse as a TBH formation mechanism starting with collisionless particles of random velocities but zero net angular momentum.

Quasar observations indicate spectra with very strong metallic absorption lines. The population II stars of the galactic centre would mainly consist of Hydrogen and Helium, which has previously troubled spheroidal BH quasar models. Because SNe are efficient at generating heavy elements, the TBH creation SN would have ejected a substantial amount (perhaps $10^6 M_\odot$) of metallic material which then shrouds the radiation of the central core, causing the absorption spectra we see.

3 STABILITY OF ROTATING TOROIDAL BLACK HOLES

For some time, following the work of Hawking (1972) and Hawking & Ellis (1973), it was thought that TBHs were unstable, albeit marginally. This somewhat contra-intuitive result assumed that Einstein's cosmological constant (Λ) was zero. Numerical computations of collisionless particles resulting in a transient

toroidal event horizon and assuming $\Lambda = 0$ were performed by Abrahams et al (1994), Hughes et al (1994) and Shapiro, Teukolsky and Winicour (1995). These results were consistent with the topological censorship theorem of Friedman, Schleich and Witt (1993) which implies that a light ray cannot pass through the central toroidal aperture before the topology becomes spherical. More recently, papers by Huang & Liang (1995); Aminneborg et al (1996); Mann (1997); Vanzo (1997) and Brill (1997) have provided mathematical descriptions of TBHs within the framework of general relativity. These equations assume that the cosmological constant is negatively valued to admit stability for the TBH and is literally constant throughout the spacetime described, which has an anti-de Sitter (AdS) background. The Vanzo paper makes it clear that a TBH can exist in a virtually flat spacetime because the TBH size is determined by the mass and conformal class of the torus, not by the cosmological constant. Rotating charged black (cosmic) strings have been described by Lemos & Zanchin (1996). A spacetime metric for a rotating, uncharged TBH presented by Klemm, Moretti & Vanzo (1998), is hereafter referred to as the KMV metric. This metric is not unique, but it is the first generalisation to admit rotation of TBHs. Holst and Peldan (1997) showed that rotating Banados-Teitelboim-Zanelli (BTZ) BHs cannot be described in terms of a 3+1 split of spacetime, instead spacetimes of non-constant curvature are required.

Physical measurements to date have been unable to establish conclusively whether Λ is positive or negative. Arguments against TBH stability have assumed that in our universe, the constant is precisely zero everywhere. The weak energy condition is assumed to be satisfied, although there are reasons to doubt its validity in extreme circumstances. Topological BHs in anti de-Sitter spacetimes are now known not to conflict with the Principle of Topological censorship, for a recent discussion, see Galloway et al (1999). Intuitively, rotating TBHs are not dissimilar to Kerr BHs in that both contain ring shaped singularities whose radii are determined by the angular momentum assuming constant BH mass. One extra parameter is necessary to characterise a stationary TBH in addition to the mass, angular momentum and charge of the Kerr-Newman metric. This parameter determines the exact geometry of the torus and can be expressed in several different ways e.g. the major radius R_1 , the minor radius R_2 the ratio of the radii R_2/R_1 (as used here) or the ratio akin to a Teichmüller parameter presented by the KMV paper. Stationarity is preserved only when this parameter achieves a balance with the TBH mass and angular momentum, and to a lesser degree the charge.

It was demonstrated by Gannon (1976) that for non-stationary BHs in asymptotically flat spacetimes, the topology of the event horizon must be either spherical or toroidal. A rotating TBH located at the centre of a galaxy surrounded by accreting matter is not stationary and so there is a possibility that the non-stationarity is sufficient to provide the necessary stability. The stationary BH metrics containing physical singularities can be misleading. For instance, the Kerr metric contains a ring singularity surrounded by vacuum i.e. a universe devoid of other matter. This is a gross simplification of what we would find in nature. Suppose a Kerr BH forms by the collapse of a rotating neutron star and that there is no other matter nearby and no matter is ejected during the implosion. The matter forming the surface of the neutron star takes a finite proper time to reach the singularity, say t_1 . On the other hand, viewed from infinity, this matter never crosses the event horizon, less still reaches the singularity. According to distant observers, the matter is trapped or frozen just above the event horizon. Also, just as the matter crosses the event horizon in finite time t_2 , it also witnesses the end of the external universe at the same moment. Obviously $t_2 < t_1$ so, the answer to the question: “when does the BH become stationary to distant observers?” is never. Indeed, one might even venture that truly stationary spacetimes are forbidden. Therefore it is dangerous for us to be guided by predictions about BH stability which rely on stationarity as one of the underlying assumptions.

Perhaps there is some deeper significance underlying the unobtainability of stationarity. Consider a closed universe approaching a big crunch and contracting rapidly in all directions. The surface defining the outer reaches of this universe could be considered as the event horizon of a BH beyond which spacetime does not exist in the usual sense. This is a BH that could conceivably approach stationarity in a short and finite time as measured by the clocks of all internal observers, there being no external observers. A singularity develops which is accessible to all the infalling matter. The outermost layers of the imploding universe catch up with the innermost layers at the Cauchy horizon, the surface of infinite blue shift. A vacuum develops in the region surrounding the singularity as it swiftly becomes devoid of matter and stationarity is achieved. The singularity now contains the entire mass of the pre-collapse universe and the Pauli exclusion principle does not participate in the physics of the singularity. What grounds are there for discarding the Pauli

exclusion principle? This principle has successfully predicted the existence of white dwarfs and neutron stars. Could quark degeneracy arise? What might string theory predict? There are obvious similarities between the Pauli exclusion principle and the premise that stationary BHs are forbidden. If it is true that BHs truly abhor stationarity then presumably re-expansion would be the only option.

Suppose that a TBH with a substantial central aperture is rotating in asymptotically flat space with a near maximal angular momentum (event horizon velocity approaching the speed of light). In principle, there is no reason why the rotational energy of this TBH cannot be arbitrarily larger than the TBH's rest mass, whereas a Kerr BH can only hold at most 29% of its total energy in rotational form, the remainder being the irreducible mass. According to topological censorship, the TBH must become spheroidal before a light ray can traverse the aperture. The fate of the excess rotational energy is something of a conundrum. Is the excess energy ejected exceedingly rapidly by some undiscovered mechanism? Is the topological censorship flawed? Would the TBH break up into multiple co-rotating spheroidal BHs? Does the Kerr hole rotate above the extremal limit, violating cosmic censorship? These problems can be circumvented for now by assuming a negative cosmological constant.

It seems somewhat coincidental that the cosmological constant is so nearly zero and not very much larger in magnitude, on purely theoretical grounds a value 120 orders of magnitude larger than observational limits might have been expected. One plausible suggestion was proposed by Coleman, 1988. According to the author, macroscopic cancellation mechanisms operate on the zero point energies under normal circumstances and these result in a zero expectation for Λ . The situation is, however, complicated in the presence of intense gravitational fields. The most intense gravitational fields we know of are caused by BHs, particularly in the immediate vicinity of the singularities residing within the event horizons. Within these intense gravitational fields, the cancellation of zero point energies operates imperfectly and gives rise to what may be considered a localised but substantial cosmological 'constant'. By this means, it is quite possible that TBH stability can be ensured even within a universe where elsewhere Λ is small. Prior to this work, it had been suspected that zero point energy might play a part in the physics of curved spacetimes because of imperfect cancellations complicating the assumptions underlying the quantum mechanical technique of renormalization (Misner, Thorne & Wheeler, 1973). It comes as little surprise that quantum mechanical effects may have a prominent role in describing the conditions where the spacetime of classical general relativity becomes singular, the stability of the TBH structure could prove to be the only direct evidence we have of this.

The KMV metric has axial symmetry. The horizons are Riemannian surfaces of constant gravity that obey the familiar BH entropy-area laws. We may utilise the membrane paradigm approach (Thorne, Price and MacDonald 1986) when considering the physics of these BHs outside the event horizon. The fact that these objects are thermodynamically well behaved, whilst interesting, is of little relevance to the present discussion. Parallels between rotating TBH solutions and the Kerr solutions for spinning spheroidal holes may be drawn, for instance both have ergoregions external to their event horizons and the maximum angular momentum of each is bounded for a given mass. Conservation of mass and angular momentum is known to be satisfied. A maximally rotating Kerr hole has a static limit extending to $2M$, double the radius of the outer event horizon. The equator of the static limit surface travels at the speed of light. The ergosphere occupies the region between the static limit and the event horizon within which everything is compelled to co-rotate with the BH due to the spacelike character of the time coordinate. Similarly, the maximal KMV metric determines the ratio r_{st}/r_+ to be 1.59. Orbits within 300% of the maximum Kerr event horizon radius are unstable and matter tends to be drawn towards the hole (the accretion disk). For a maximally rotating BH, entering the ergosphere becomes impossible because incoming particles would have to travel faster than light and possess an infinite amount of energy. Similarly, if the hole is rotating slightly slower than this then only a tiny fraction of the external particles will penetrate the ergosphere, those with the most extreme kinetic energies.

Let us now consider the ergosphere enshrouding a TBH event horizon as identified by the KMV metric. Fig.3 depicts a cross-sectional view of a rotating toroidal hole. The event horizon will be enshrouded by an ergoregion which, depending upon the precise geometry of the TBH, might entirely seal the central aperture. Beyond the ergoregion lies what is sometimes referred to as a zone of unstable orbits within which particles are unable to establish repeating orbital patterns by following geodesic pathways. The ergoregion does not intersect the event horizon at any point, as it does at the poles of the Kerr holes. Particles cannot penetrate a maximally rotating TBHs ergoregion, whichever point of the ergoregion's surface they are aimed at. The

maximal rotation rate will not be achieved in practice because the BH is able to reduce its rotation rate by several methods which are relevant to jet formation and several theoretical reasons such as the fact that the internal singularity would become naked, even as viewed from infinity.

4 METRIC OF ROTATING TOROIDAL BLACK HOLE.

The KMV metric for an uncharged rotating TBH in asymptotically anti de-Sitter (AdS) spacetime is known to be non-unique and is tentatively forwarded as a model for the naturally occurring TBH. The primary reservations concerning the physical applicability of the topological BH metrics in AdS gravity are that Λ is assumed to be independent of location and, contrary to the most reliable observations, negative in value. Given the uncertainty surrounding the role of quantum gravity in BH physics, these assumptions may be invalid. The metric describes a vacuum solution of Einstein's equation which has reached equilibrium after an infinitely long (in coordinate time) collapse phase. It possesses a ring singularity, but, no provision for accreting matter has been made. Indeed, a massive accretion disk may act as a stabilising influence on a TBH within an asymptotically flat spacetime. A negative cosmological constant may be thought of as contributing a cosmological attraction. In its absence, the combination of a host galaxy's matter and the nearby massive accretion disk surrounding the outer periphery of a TBH located within a galactic nucleus may provide a natural substitute for the stabilising negative Λ used in the AdS metrics.

With these considerations in mind, let us now turn our attention to the KMV metric which, for convenience, is now recalled:-

$$ds^2 = -N^2 dt^2 + \frac{\rho^2}{\Delta_r} dr^2 + \frac{\rho^2}{\Delta_P} dP^2 + \frac{\Sigma^2}{\rho^2} (d\phi - \omega dt)^2 \quad (16)$$

where

$$\rho^2 = r^2 + a^2 P^2 \quad (17)$$

$$\Sigma^2 = r^4 \Delta_P - a^2 P^4 \Delta_r \quad (18)$$

$$\Delta_r = a^2 - 2mr + r^4/l^2 \quad (19)$$

$$\Delta_P = 1 + \frac{a^2 P^4}{l^2} \quad (20)$$

$$N^2 = \frac{\rho^2 \Delta_P \Delta_r}{\Sigma^2} \quad (21)$$

$$\omega = \frac{\Delta_r P^2 + r^2 \Delta_P}{\Sigma^2} a \quad (22)$$

Here, the angular velocity is ω , the equatorial angle is ϕ , P is another angular variable with some period T , r is a pseudo-radial coordinate, a is the angular momentum per unit mass and l is defined as $\sqrt{-3/\Lambda}$. The ratio of T to 2π is analogous to the Teichmüller parameter describing a flat torus in Riemannian geometry. TBH mass by the ADM definition is $M = mT/2\pi$ and angular momentum $J = Ma$. The coordinate r , as in the Boyer-Lindquist form of the Kerr metric, is only a true radial coordinate as $r \rightarrow \infty$ and $r = 0$ is the location of a ring singularity not corresponding to zero radius. Unlike the Kerr case, as $r \rightarrow -\infty$ within the equatorial plane, this point lies outside the event horizon. It would be preferable to introduce a coordinate transformation whereby $r' = f(r)$ such that $f(\infty) = \infty$, $f(0) = a$ (say) and $f(-\infty) = 0$ and select $f(r)$ such that r' is an affine parameter, but this is beyond the scope of the present discussion.

In order for the metric to describe a torus, P is a periodic variable with period T and is covered by four patches $P = \lambda \sin \vartheta$ at $[\vartheta = 0, \vartheta = \pi]$ and $P = \lambda \cos \vartheta$ at $[\vartheta = \pi/2, \vartheta = 3\pi/2]$ where λ is a constant such that $T = 2\pi\lambda$. Between these points the behaviour is defined by $\cos \vartheta$ being some C^∞ function (infinitely differentiable) of $\sin \vartheta$ and vice versa.

Upon inspection of the metric equation (16) we see that Δ_r in equation (19) becomes zero at the event horizon. Inner and outer event horizons exist as real and positive roots of the quartic equation with real coefficients: $r^4 - 2ml^2r + a^2l^2 = 0$ along with two other physically less meaningful complex conjugate roots. For $a = a_c = \sqrt{3} \times \sqrt[3]{lm^2/4}$ (the extremally rotating case) these two roots coincide and it is straightforward to verify that the real roots are $r_{\pm} = \sqrt[3]{ml^2/2}$.

The ergosphere is defined as the region between the outer event horizon and the static limit hypersurface at which the metric coefficient of dt^2 vanishes, i.e. $g^{tt} = \omega^2 \Sigma^2 / \rho^2 - N^2$ which is solved for r by another quartic $r^4 - 2ml^2r - a^4P^4 = 0$. This polynomial has one real and positive root, one real and negative root and two complex conjugate roots. The real positive root has a minimum value of $\sqrt[3]{ml^2}$ for $P = 0$ which is larger than r_+ by a factor of 1.59. The static limit hypersurface is well separated from the event horizon and, unlike the poles of the Kerr situation, these surfaces are nowhere contiguous. We therefore observe a substantial ergoregion.

Examining the equatorial plane by setting $dP = P = 0$ the metric reduces to:

$$ds^2 = - \left(\frac{r^2}{l^2} - \frac{2m}{r} \right) dt^2 + \left(\frac{r^2}{l^2} - \frac{2m}{r} + \frac{a^2}{r^2} \right)^{-1} dr^2 + r^2 d\phi^2 - 2ad\phi dt \quad (23)$$

In order to determine the trajectories of null geodesics within this hypersurface use can be made of the Euler-Lagrange equations where we set $K = ds^2/2$: -

$$\frac{\partial K}{\partial x^a} - \frac{d}{du} \left(\frac{\partial K}{\partial \dot{x}^a} \right) = 0 \quad (24)$$

where the overdot denotes differentiation with respect to some affine parameter, u , we find that the equatorial metric, equation (23), may be partial differentiated with respect to t and ϕ respectively and integrated with respect to u to give: -

$$\left(\frac{2m}{r} - \frac{r^2}{l^2} \right) \dot{t} - a\dot{\phi} = \alpha \quad (25)$$

$$r^2 \dot{\phi} - a\dot{t} = \beta \quad (26)$$

Where α and β are constants of integration. We may define a third constant as $\gamma = \alpha/\beta$ and use it to relate both equations. We then obtain: -

$$\left(\frac{2m}{r} - \frac{r^2}{l^2} \right) \dot{t} - a\dot{\phi} = \gamma(r^2 \dot{\phi} - a\dot{t}) \quad (27)$$

which upon rearrangement reads: -

$$\frac{d\phi}{dt} = \frac{\dot{\phi}}{\dot{t}} = \frac{2ml^2 - r^3 + \gamma arl^2}{arl^2 + \gamma l^2 r^3} \quad (28)$$

Next, boundary conditions are imposed by considering the extremal case $a = a_c$ for which the angular velocity at the event horizon r_+ is, using the expressions for a_c and r_+ and noting that $\omega = a/r^2$ in the equatorial plane.

$$\left. \frac{d\phi}{dt} \right|_{r=r_+} = \Omega_H = \frac{a_c}{r_+^2} = \frac{\sqrt{3}}{l} = \frac{2ml^2 - r_+^3 + \gamma ar_+ l^2}{ar_+ l^2 + \gamma l^2 r_+^3} \quad (29)$$

Some algebra reveals that the constant of proportionality is: -

$$\gamma = \frac{3\sqrt[3]{2}ml - 2\sqrt{3}a\sqrt[3]{ml^2}}{\sqrt{3}\sqrt[3]{2}ml^2 + 2al\sqrt[3]{ml^2}} = \frac{3r_+^2 - \sqrt{3}al}{\sqrt{3}lr_+^2 + al^2} \quad (30)$$

So now the rate of change of ϕ with respect to coordinate time t is fully determined. Following straight from the metric and the condition that $ds = 0$ for null geodesics we obtain: -

$$- \left(\frac{r^2}{l^2} - \frac{2m}{r} \right) \dot{t}^2 + \left(\frac{r^2}{l^2} - \frac{2m}{r} + \frac{a^2}{r^2} \right)^{-1} r^2 + r^2 \dot{\phi}^2 - 2a\dot{\phi}\dot{t} = 0 \quad (31)$$

Dividing throughout by $(dt/du)^2$ eliminates the affine variable and we get an expression for dr/dt in terms of the previously derived expression for $d\phi/dt$: -

$$\frac{dr}{dt} = \sqrt{\left[2a\left(\frac{d\phi}{dt}\right) - r^2\left(\frac{d\phi}{dt}\right)^2 + \frac{r^2}{l^2} - \frac{2m}{r}\right] \left[\frac{r^2}{l^2} - \frac{2m}{r} - \frac{a^2}{r^2}\right]} \quad (32)$$

It would be possible to continue this analysis by integrating with respect to t for each variable r and ϕ , but the expressions would not be particularly elegant. It is sufficient for now to say that these equations allow the null congruences of the equatorial plane to be readily determined by numerical methods.

5 ROTATING TOROIDAL BLACK HOLE IN ASYMPTOTICALLY FLAT SPACE.

In order to visualise a TBH in asymptotically flat space and its effect on the surrounding spacetime, a method for approximating the time dilation of the space surrounding arbitrarily complex mass configurations will be introduced. We begin by deriving the time dilation within a Schwarzschild spacetime which has the metric:-

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2(d\vartheta^2 + \sin^2 \vartheta d\phi^2) \quad (33)$$

The event horizon of this static spacetime occurs when g_{rr} becomes infinite, or $r_+ = 2m$ in geometrized units. We now consider the time dilation of a stationary particle located at some constant r , ϑ and ϕ . The metric interval ds can be interpreted as the proper time of particles travelling on timelike paths so we take $d\tau = ds$. The time dilation may be read from the metric at once as: -

$$\frac{d\tau}{dt} = \sqrt{\left(1 - \frac{2m}{r}\right)} = \sqrt{\left(1 - \frac{r_+}{r}\right)} = \sqrt{\psi} \quad (34)$$

where

$$\psi = \left(1 - \frac{2m}{r}\right) \quad (35)$$

As $r \rightarrow \infty$ we notice $d\tau/dt = 1$ whilst $d\tau/dt$ decreases towards zero as the event horizon is approached, as we would expect. Now we shall permit the particle to undergo radial motion $dr \neq 0$, $d\vartheta = d\phi = 0$. The metric is divided throughout by dt^2 and we identify the particle's radial velocity in local coordinates as being $v_p = dr/d\tau$. This yields a similar expression to the last but with the introduction of a v_p dependent term: -

$$\frac{d\tau}{dt} = \frac{\psi}{\sqrt{\psi + v_p^2}} \quad (36)$$

For the Schwarzschild BH, the event horizon and stationary limit coincide at $r = r_+$ and $d\tau/dt$ becomes zero there. A radial velocity can affect the time dilation but cannot alter the location of the hypersurface at which the time dilation approaches zero. Conversely, in the Kerr case which we now briefly address, $d\tau/dt$ becomes zero for stationary particles outside the event horizon on the stationary limit, the outermost boundary of the ergosphere. Particles motionless with respect to distant observers will appear to freeze at the static limit but this is not so for particles rotating with the Kerr hole, they can both penetrate and escape the ergosphere in a finite coordinate time. Particles counter rotating with the hole will find that the time dilation becomes zero at radii beyond the static limit. Clearly the location of the stationary limit is meaningful only for particles with zero coordinate velocity. We could think of the ergosphere as being a zone where some particles are able to travel on spacelike trajectories with most spacelike trajectories close to the event horizon. Whereas negative energy states are only available within the ergosphere of a Kerr BH, a charged Kerr-Newman BH offers negative energy states beyond the static limit. When viewed in this way, using the static limit to define the extent of the ergosphere seems rather arbitrary.

Returning our attention to the Schwarzschild metric, we examine the situation where the particle undergoes transverse (azimuthal) motion with $dr = d\phi = 0$ noting that $v_p = r^2 d\phi/d\tau$ this leads to: -

$$\frac{d\tau}{dt} = \sqrt{\frac{\psi}{1 + v_p^2}} \quad (37)$$

The purpose of this analysis is to arrive at expressions that describe the time dilation experienced by observers nearby a moving point mass where the clock at infinity is motionless relative to the nearby observers. The Schwarzschild coordinates do not allow us to do this directly since the mass of the singularity is stationary with respect to observers at infinity. By taking the limit as $m \rightarrow 0$ we obtain $\psi \rightarrow 1$ and both the previous equations reduce to the time dilation of special relativity when two objects are in relative motion. We will use these limits to introduce a contribution to the time dilation equivalent to inducing a motion of the clocks at infinity. The situation then describes clocks at infinity moving with velocity v_p , clocks of local observers moving with velocity v_p and a motionless point mass. Since all inertial frames are equivalent, we may think of this as stationary clocks and a moving mass with velocity v_p in the opposite direction. The various possibilities are depicted in Fig.4. Condition 6 has been determined by taking the ratio of the expression in condition 4 with the expression in condition 2. Likewise, condition 7 has been determined by taking the ratio of the expression in condition 5 with the expression in condition 3. By taking these ratios, we are applying boosts which remove the time dilation contributions of expressions 4 and 5 which were purely due to the relative motions of the clocks. We are then left with motionless clocks in the presence of a moving point mass.

We see that the expressions in conditions 2 and 3 are identical implying that time dilation between observers in the absence of gravity is independent of the direction of motion. Conditions 2 and 3 are limiting cases of conditions 4 and 5 respectively in the absence of matter. Because the expressions of conditions 1 and 7 are identical we may conclude that only the component of the mass's velocity towards the local clock (not the clock at ∞ since this is always unaffected by the mass) contributes to the time dilation of the local clock relative to the clock at infinity. Condition 6 can then be used to calculate the time dilation precisely in more general circumstances providing that v_p is the velocity component towards the local clock. Note also that there is no requirement for the clocks and the mass to be aligned as they are in Fig.4, the expressions presented are valid for all configurations due to the symmetries of the Schwarzschild geometry.

Suppose that some point singularity is subdivided into N smaller but not necessarily equal masses and let us allow these point masses to be located at the same point in space as the parent singularity. In order to recover the correct expression for time dilation given in equation (34), we are obliged to perform N summations of the ratios r_+/r where r_+ relates to the Schwarzschild radius of the mass of each child singularity in turn according to the equation $r_+ = 2m$. Now we may generalise this for the purposes of approximation such that the point masses are not coincident but are located separately in space. Thus the distance r will in general be different for each point mass. Restoring natural constants, an expression of the following form is useful:

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2G}{c^2} \times \sum_n \frac{M_n}{R_n}} \quad (38)$$

This may be thought of as a pseudo-principle of superposition. Let us now consider how these results may be used to approximate an asymptotically flat spacetime containing a ring shaped singularity. Firstly we confine our discussion to a momentarily stationary ring shaped singularity i.e. one with zero angular velocity and a radius R_1 whose derivative with respect to time is momentarily zero. We will derive an expression for the time dilation relative to observers at spatial infinity experienced by a motionless stationary observer due to the momentarily motionless ring singularity. We assign the ring singularity a constant mass per unit length, b , and radius, a , such that the ring's mass is $2\pi ab$ and consider the time dilation within the plane of the ring. By symmetry, the only independent coordinate is the radial one r , and the time dilation $d\tau/dt$ at that point is approximated by the expression:-

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2abG}{c^2} \int_0^{2\pi} \frac{d\phi}{\sqrt{r^2 + a^2 + 2ar \cos \phi}}} \quad (39)$$

Setting $\phi = \phi/2$ the time dilation can be expressed in terms of a complete elliptic integral of the first kind, $K(k)$:

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{8abG}{(a+r)c^2} \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}}} = \sqrt{1 - \frac{8abG}{(a+r)c^2} K(k)} \quad (40)$$

where

$$k = \frac{2\sqrt{ar}}{a+r} \quad (41)$$

The point at the centre of the ring $r = 0$ is a special case which is readily integrated to give: -

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{4\pi bG}{c^2}} \quad (42)$$

In order to describe a TBH, the ring density b must be smaller than $c^2/4\pi G$. By writing $(\sqrt{a} - \sqrt{r})^2 \geq 0$ and expanding we find that $k \leq 1$ in all cases of equation (41) satisfying the requirements of the elliptic integral, k being the ratio of the geometric and arithmetic means of the constants r and a . For the static case, the term within the square root becomes zero at the event horizon. If rotation is introduced, the time dilation will become infinite not at the event horizon but at the static limit, the external boundary of the ergosphere where the invariant interval of stationary particle pathways is lightlike.

Next, we allow the ring singularity to rotate with constant angular velocity ω so that the velocity of a point on the ring is $v_r = a\omega$. We consider the component of this velocity directed towards the observer P situated at some radius r from the centre of the ring and, once more, within the plane of the ring (the equatorial plane). This component will contribute to the time dilation experienced by the observer according to the expression presented in condition 6 of Fig.4. Once more we sum over the terms within the square root causing a deviation from parity of proper and coordinate time. Due to the lack of a rotating TBH metric in asymptotically flat space, Newtonian approximations will be used to select values for the ring's angular velocity such that a constant ring radius is maintained through the balance of centripetal and gravitational forces. Once again, the ring's mass is $2\pi ab$ and the centre of mass is concentrated at the centre of the ring so the ring velocity is defined in terms of the ring's mass per unit length and radius as being: $(a\omega)^2 = 2\pi bG$. Recalling the expression for time dilation of condition 6 and substituting $\psi = 1 - (2m/r) = 1 - r_+/r$ then rewriting in such a way as to give a separate and integrable deviation from unity within the square root we get: -

$$\frac{d\tau}{dt} = \psi \sqrt{\frac{1+v_p^2}{\psi+v_p^2}} = \sqrt{1 - \left(\frac{r_+}{r^2}\right) \left[\frac{r(1+2v_p) - r(1+v_p^2)}{1+v_p - r_+/r} \right]} \quad (43)$$

Taking the limit $r \rightarrow r_+$ we get $d\tau/dt = \sqrt{v_p^2 - 1}$ which means only particles travelling at the speed of light can remain on the horizon, as expected. As it stands, this formula allows the deviation from unity within the square root to be summed for an arbitrarily large number of point masses, regardless of the mass contained by each. Simplification is possible if we assume that all these point masses are infinitesimally small so that the Schwarzschild radius of each is negligible compared to the distance between each mass and the local clock where $d\tau/dt$ is to be determined, $r_+ \ll r$. Implementing this simplification and including the integration symbol to emphasise the fact that the point masses should be vanishingly small, we obtain: -

$$\frac{d\tau}{dt} = \sqrt{1 - \int \frac{r_+}{r} \left(\frac{1+2v_p}{1+v_p} \right)} \quad (44)$$

We now derive expressions for the time dilation relative to observers at spatial infinity for points surrounding the rotating ring shaped singularity. These points are assigned cylindrical coordinates r, z where by symmetry the ϕ coordinate is redundant. This time we are not confined to the equatorial plane. As we would expect, the integral resembles the previously derived equation containing an elliptic integral but with additional complexity: -

$$\frac{d\tau}{dt} = \left[1 - 2ab \int_0^{2\pi} \frac{\sqrt{a^2 + r^2 + z^2 - 2ar \cos \vartheta} + 2ar\omega \sin \vartheta}{\sqrt{a^2 + r^2 + z^2 - 2ar \cos \vartheta} (\sqrt{a^2 + r^2 + z^2 - 2ar \cos \vartheta} + ar\omega \sin \vartheta)} d\vartheta \right]^{1/2} \quad (45)$$

Note that the time dilation at the centre of the ring ($r = 0, z = 0$) is still given by equation (42) because the velocity of each point mass is perpendicular to the line connecting the point mass to the local observer.

It would be possible, but more complicated, to determine the location of event horizons using a similar method. One would need to transform the local observers to those of a locally non rotating frame (LNRF). This would be achieved in the equatorial plane by equating the angular velocity of the ring with that of the local observers. Starting with condition 6 of Fig.4, one would generalise to the case where the local clock and point mass have separate (non-zero) velocities with respect to the clock at spatial infinity by applying a Lorentzian boost to the coordinate clock. Then, equivalents to the expression in equation (43) and equation (44) would need to be found.

The time dilation can be computed numerically but care is needed when selecting the ring's angular velocity, ω otherwise the situation becomes unphysical due to frame dragging velocities in excess of c . This formula was used to determine the shape of the ergosphere in Fig.3 when viewed in cross section. Fig.5 presents a 3-dimensional projection of the time dilation as viewed by observers located at spatial infinity for the equatorial plane intersecting a rotating TBH. This embedding diagram portrays local time dilation (as viewed by distant observers) due to the presence of mass as the deviation from an otherwise flat plane according to equation (45). The hole drags local spacetime with it in synchrony with the event horizon. Accordingly, inertial particles travelling within the equatorial plane along initially radial geodesics from spatial infinity are compelled to orbit the TBH until their angular velocity reaches that of the event horizon. This occurs at the moment the horizon is crossed. Shading is used to denote the angular velocity of locally non-rotating observers as measured by distant observers, the darker shades representing angular velocities approaching that of the TBH. A section of the outer funnel has not been plotted to provide visibility of the TBH aperture region. The ergoregions have not been identified here.

6 JET FORMATION FROM TOROIDAL BLACK HOLES.

The near-maximally rotating TBH undergoing accretion provides an excellent mechanism for the formation of ultra-relativistic (Lorentz factor typically 5 to 10) bi-directional jets as have been observed in quasars. The purpose of this section is not to explore the behaviour of the jets as they travel towards the distant radio lobes, the magnetohydrodynamics of which has been studied in great detail elsewhere, nor to analyse the myriad of particle interactions capable of extracting rotational energy from the TBH ergosphere. Rather, the essential differences between existing models and the accretion of matter on to a rotating TBH shall be outlined. Supermassive BHs have long been thought to reside at the heart of quasars and active galactic nuclei. These objects have estimated masses of the order of one billion suns ($10^9 M_\odot$). A typical rotating galaxy possesses ample angular momentum to spin up a toroidal hole of such mass to near maximal rotation. An upper limit on rotational velocity exists because the peak velocity of the event horizon cannot exceed that of light. In practice, the maximum rotation rate will not be reached because the hole will increasingly radiate energy through gravitational waves and because at the maximum rate, the internal singularity would be revealed. We will investigate how the jets are also able to transport angular momentum away from the TBH.

A young galaxy may harbour a TBH whose spin rate is increasing. Once the spin rate has reached its peak, after a short (in cosmological terms) delay, equilibrium is achieved and the accretion process is balanced by the angular momentum released by the hole due to gravitational radiation, jet formation processes and growth through capture of mass and angular momentum. Of these, the outflow of angular momentum is typically dominated by jet generation processes. This mechanism maintains a rapidly rotating TBH, but means that the accreting matter is rotating more rapidly than the space surrounding the hole. Essentially, the interaction between this accreting mass and the enormous flywheel of the rotating TBH constitutes the mechanism for energy release along jet pathways. Fig.6 illustrates a rotating TBH surrounded by an accretion disk. Since the TBH is able to shed any excess angular momentum by several mechanisms, its angular velocity is below that of the accretion disk $\omega_{TBH} < \omega_{DISK}$. One could say that the TBH is almost unaffected during the process (apart from gradual mass accumulation), acting somewhat like a catalyser for the ejection of angular momentum along the jets. Jet formation can therefore progress for substantial periods of time.

At the central aperture, spacetime is dragged around with the hole. The central aperture is a negative gravitational potential well, a gravitational vortex containing deeply negative energy states. Matter that passes through this central aperture will be obliged to travel along geodesics which appear to the external universe to be rotating. Particles capable of escaping to infinity have to move at relativistic velocities closely

aligned with the spin axis. Matter is able to travel in either direction along the rotational axis in order to achieve this, and angular momentum is transported away from the TBH equally by each jet. We focus our attention here on a TBH whose central aperture is small and thus more likely to provide powerful and narrow jets.

Matter travelling through the aperture will undergo gravitational slingshot and could be propelled outwards along the jets, however the primary importance of this process will be to cause frictional heating due to high speed collisions of the accreting matter within the central aperture. This accreting matter would be heated to at least several million degrees (and perhaps as much as $10^{11} K$) by these collisions, transforming the matter into a plasma emitting X-rays and some gamma rays.

Ejection mechanisms such as the Penrose process (Penrose, 1969), superradiant scattering (photonic counterpart of the Penrose process) and their analogues (e.g. due to particle collisions) will dominate. For convenience, the term Penrose process is used loosely to refer to all variations. The Penrose process explains how a rotating BH can release energy by virtue of negative energy orbits within BH ergoregions. A particle travelling through the ergosphere might disintegrate into two particles, one of which travels rapidly towards the event horizon and be captured by it whilst the other may be ejected to infinity along one of the jet paths with more energy than the original particle had. The Penrose process efficiency improves if the particles have relativistic incident velocities, particularly those opposing the hole's rotation. A physical example of particle disintegration is when atoms fragment within a high energy plasma resulting in the release of a free electron and an ionized atom. More generally, ergoregional particle-particle collisions in which angular momentum and total energy are conserved may cause one of the resultant particles to be ejected to infinity (Piran, Shaham & Katz, 1975; Piran & Shaham, 1977). Particles are always ejected in a way that reduces the BH's angular momentum and rotational energy which increases the surface area of the event horizon, in keeping with the entropy law. As has been described by Wagh, Dhurandhar & Dadhich, 1985 and Bhat, Dhurandhar & Dadhich, 1985, the presence of an electromagnetic field and/or charged particles (such as within a high temperature plasma) can dramatically increase the efficiency of the Penrose process, easily to a level where net rotational energy may be extracted from the hole. The astrophysical significance of such processes has been controversial up to now because realistic spherical BHs possess negligible charge and there is no reason to suppose that strong magnetic fields would be present. The situation is dramatically altered for a rotating TBH, though. Circulating charge can accumulate which induces a strong toroidal magnetic field.

According to the membrane paradigm, we may think of the TBH's (infinitesimally stretched) event horizon as being an electrically conducting surface where electric fields incident to the surface are terminated by an appropriate surface electric charge density. Also, the surface current will be such that the magnetic field parallel to the surface is terminated, in this way there will be no parallel magnetic field inside the event horizon. The effective surface resistivity will be of the order of several hundred ohms. The accreting matter contains neither magnetism nor net charge initially. The hole is, however, spinning and dragging local spacetime around with it. The charged particles entering the ergoregions are mainly electrons and protons. Low efficiency Penrose processes will preferentially eject particles of larger charge/mass ratios (the electrons) and a net positive circulating charge will emerge hovering above the horizon. The toroidal membrane rotates and drags these positive charges around with it thus forming a circular electrical circuit. Current flowing in the circuit gives rise to an axial magnetic field through the central aperture, modulating the efficiency of particle emissions via the Penrose process thereby reinforcing the circulating charge and magnetic field. This magnetic field also plays a role in collimating the jets as they are launched, with charged particles spiralling along the magnetic field lines generating synchrotron radiation. Similarly, the toroidal magnetic field channels free charged particles from the outer accretion disk into the central aperture, spiralling along the lines of magnetic flux.

The form of the magnetic field surrounding a conducting toroidal shell is shown in Fig.7. Magnetic field lines are illustrated which arise when a current circulates around the toroidal shell. The field lines thread the toroidal aperture. Because the aperture can become arbitrarily small, the magnetic field can become arbitrarily large within this region. Computed plots of magnetic field strength along the equatorial plane of the torus are given for four separate toroidal geometries in Fig.8(a). These have been calculated using the usual Biot-Savart relations and assume a constant and uniform surface current density J . Analytically, the magnetic field strength perpendicular to and within the equatorial plane at some displacement from the axis a , making use of symmetry, is given by the double integral:-

$$\vec{B}(a) = \int_0^{2\pi} \int_0^{2\pi} \frac{\mu_0 J t (a \cos \phi - t \cos 2\phi)}{4\pi(a^2 + t^2 + r_2^2 \sin^2 \vartheta - 2at \cos \phi)^{3/2}} d\phi d\vartheta \quad (46)$$

where

$$t = r_1 + r_2 \cos \vartheta \quad (47)$$

The Biot-Savart law simplifies at the centre of a circular current loop carrying a current I and it is straightforward to verify that the field strength at that point is: -

$$\vec{B}(0) = \frac{\mu_0 I}{2r_1} \quad (48)$$

Nearby the toroidal surface, Ampere's circuital law, equation (49), states that the current enclosed by a closed path determines the sum of the magnetic field along the same closed path so the field strength is always finite at the shell's surface.

$$\oint B ds = \mu_0 \times I \quad (49)$$

The same law tells us that the integral, equation (46), is independent of r_2 providing $|a| < (|r_1| - |r_2|)$ or $|a| > (|r_1| + |r_2|)$, therefore some simplification is available by setting $r_2 \rightarrow 0$ whilst maintaining a constant circulating current. The integral of equation (45) is not analytic but can be expressed in terms of multiple elliptic integrals. Numerical computations have been used to derive plots in Fig.8(a) which clearly show that the axial field within the central aperture is stronger than in the region surrounding the outer periphery of the torus as measured by local inertial observers. This is particularly true for the geometries where $r_2 \rightarrow r_1$ that would produce the narrowest jets. The charged particles spiral around the strong field lines of the aperture achieving high velocities and alternate between contra-rotation and co-rotation during each cycle of their spiral. During the contra-rotation phase, they are particularly likely to participate in particle collisions in which energy and momentum is radiated to infinity at the expense of the angular momentum of the TBH.

To better approximate the field surrounding a TBH we must also consider the gravitational time dilation as discussed previously. Restricting ourselves to an analysis of the time dilation generated by a momentarily static ring shaped singularity whose event horizon is transiently toroidal, we insist that the horizon coincides in the equatorial plane with the surfaces of the electrically conducting toroidal shell. Essentially, this is achieved by precise adjustment of the radius of the ring singularity ($R_{ring} > R_1$) and the ring singularity's mass per unit length. The magnetic field strength plots of Fig.8(a) are then recalculated, this time with the field strength divided by the local time dilation as viewed by a distant observer. The situation is analogous to earth based quasar observations because this magnetic field directly modulates the non-thermal radiation emanating mainly from within the TBH ergoregion. Note also, the negative energy states within the ergoregion of the central aperture will be more negative than elsewhere within the ergoregion but that the increased time dilation counteracts this. Results are presented in Fig.8(b). Again, the electrical surface current density J has been held constant but the TBH mass is different in each case. What we witness once more, this time somewhat less markedly, is that the magnetic field is still strongest within the central aperture for all cases. As the aperture becomes narrower, the ratio of peak field strength within the aperture to peak field strength external to the TBH outer periphery also increases. Numerical values for these magnetic field strength ratios are (1.86, 1.72, 1.33, 1.17) for $R_2/R_1 = (0.2, 0.4, 0.6, 0.8)$ respectively.

These facts would lead us to predict that the Penrose process will occur predominantly within the central aperture of the TBH, and less so in the outer regions. Ergoregional particle emissions in the outer regions are largely reabsorbed by collisions with the accretion disk, whereas the rarefied central aperture allows relatively unimpeded passage to scattered particles. Thus, the most energetic TBHs will be those with tightly focused jets.

Intrinsically stronger magnetic fields are to be anticipated in the vicinity of a TBH as compared to the rotating spheroidal hole situation. In order to sustain the magnetic field, electrical currents are necessary which are terminated by the membranous horizon in the case of the spheroidal hole. Replenishment of charged particles external to the BH providing the electrical current is therefore necessary. Termination of electrical currents is not mandatory in the same sense for the TBH case because electrically charged

particles may circulate parallel to the horizon. The presence of the magnetic field and the plasma gives rise to a force-free magnetosphere within the TBH's central aperture providing the plasma is sufficiently rarefied. The accumulation of circulating positively charged particles frozen near the TBH event horizon is practically identical to the situation where the TBH itself is charged.

Energetic photons (X-rays and gamma rays) generated by the plasma of the central aperture would be emitted in all directions. Ergoregional processes would be capable of ejecting them because they are travelling at the speed of light, many of them against the rotation of the TBH as viewed by locally non-rotating observers. Ejected photons would be promoted to higher energies and could inhabit the gamma ray spectrum at energies as high as the TeV range. The Penrose process will also act on the high velocity electrons and atomic nuclei of the high-energy plasma occupying the ergosphere of the aperture. All of this is accomplished at the expense of the rotational energy of the TBH. Bi-directional jets with relativistic velocities are therefore likely to result. The ejected matter and radiation necessarily must emerge from either side of the central aperture, where it begins its journey along one of the two jets. Depending on the geometry of the TBH, the jets could be tightly focused and penetrating or, conversely, spluttering weakly over a broad solid angle. Particles are preferentially ejected in close alignment with the modulating magnetic field, in this case along the spin axis of the TBH. The toroidal magnetic field provides for deeper negative energy states within the ergosphere whilst extending the region of occurrence well beyond the static limit surface. Bhat, Dhurandar and Dadhich demonstrated that when charged particles are involved in Penrose process interactions there exists virtually no upper bound on the efficiency of energy extraction.

The jets are to some extent able to self-collimate themselves if they are sufficiently focused at the source by trapping the magnetic field within themselves. Magnetohydrodynamic studies have had much success in explaining the processes involved in these normally supersonic outflows which can travel distances of several millions of light years before they begin to disintegrate. The knots visible along these jets are readily interpreted as the result of substantial short-term matter ingestion as in the case of stellar collisions with the TBH. The rotating jets will cause a net outflow of angular momentum from the TBH, which, along with energy lost by the hole due to the radiation of gravitational waves is counterbalanced by the net inflow of angular momentum due to accretion around the TBH's periphery. The reason that the jets possess angular momentum is that the particles ejected from the ergoregions are rotating with the hole and are launched by the Penrose process within the ergosphere in such a way as to generate a decelerational torque (recoil) on the TBH. The work has been achieved at the expense of the energy stored in the rotation of the TBH. By the processes described, a significant portion of the mass and kinetic energy of accreting matter and radiation is available for jet production. For detailed analysis, numerical simulations will be required.

The Penrose process reaches maximum efficiency when one of the particles heads directly towards the event horizon along the shortest path (i.e. it has the most negative energy state possible). Similarly, when the negative energy state arises due to the presence of charge on a particle, we would also expect one of the particles emerging from the collision to scatter towards the event horizon along the shortest path. When we consider the trajectory of the other scattered particle for the purely gravitational Penrose process, the particle will head directly away from the event horizon, which for the TBH central aperture is typically a poor escape route. The situation when electromagnetic interaction dominates the Penrose process is different. The potential energy of a charged particle within an electric field should be considered. In order to recoil with maximal energy extraction, the charged particle will follow a path that leads towards greatest electrical potential which, for a charged and rotating TBH is aligned axially with the magnetic field. These ejected particles will frequently collide with the accretion flow streaming from the outermost periphery of the TBH. The jets will be sufficiently strong to overcome this inward accretion flow in the regions nearest the spin axis. An almost identical scenario was analysed by Blandford and Rees in 1974 wherein relativistic plasma escapes anisotropically through orifices punctured in a cool surrounding gas and collimated beams of plasma result.

By comparison with quasar observations, we now see that the TBH model can address the issue of high energy gamma rays (20GeV to 1TeV) within the jets, these being generated almost entirely by Penrose process photon promotion. The variation in jet dispersion angles is related to the R_2/R_1 ratio of the TBH and is naturally accounted for by the TBH model. Quasar spectra can contain three separate red-shift sections, a TBH would also have at least three. These are to be anticipated because:-

- (i) The plasma of the central aperture is buried in a gravitational well.

- (ii) Jets travel relativistically in opposite directions (one of which is usually not directly detectable).
- (iii) Radiation passes through the metal rich clouds generated by the SN of the TBH creation event.
- (iv) The remoteness of the QSO galaxy correlates to a cosmological red-shift.

A carefully considered study of gravitating fluids (Marcus, Press & Teukolsky, 1977) reveals a bifurcation from the Maclaurin ellipsoids to lower energy state 'Maclaurin toroids' at high angular momenta which, the authors suggest, may be stable against all small perturbations. Alternatively, toroidal density distributions may spontaneously arise within spheroids during gravitational collapse. Butterworth & Ipser (1976) demonstrated that ergoregions can form when relativistic stars are rotating rapidly although absolute event horizons are absent. The astrophysical significance of rotating toroidal neutron stars is that they possess many similarities with the TBH of quasars. Several possibilities are now outlined.

Neutron rings possessing net charge can generate immensely strong axial magnetic fields. When ergoregions are present and accreting matter is available, bidirectional jets could arise due to the proposed quasar mechanism. Given a steady supply of material, as from a binary companion star, microquasar behaviour could result. Certain explosive Type II SNe may occur when a neutron ring forms during the collapse phase. Gamma ray bursts would be anticipated for metastable microquasars. In such cases, a white dwarf accretes matter from the companion star, the core collapses to a neutron ring at which time a brief but intense outburst of mass and angular momentum return the star to its original white dwarf state.

7 EVOLUTION BEYOND OPTICALLY BRIGHT QSO PHASE.

It is well documented that quasar activity in the early universe seemed to peak at co-moving red-shifts of $z \sim 2.5$. What is not well understood according to existing models reliant on a central supermassive spheroidal BH is why the optically bright quasar behaviour terminates so abruptly at lower red-shifts. This is particularly puzzling given that the BHs themselves cannot have altered other than gain yet more mass from their surroundings. Ideas have been proposed such as the back-reaction on the accretion flow arising from the BH once the luminosity exceeds a certain value. The accretion rate would then be abruptly reduced, and quasar activity would cease. The proposal is fundamentally at odds with intuition. Even if we suppose that the accretion rate can be steadied by this method, we would surely expect occasional stellar captures with collisional velocities sufficient to re-establish brief quasar behaviour. Evidence for such occurrences is absent.

According to the present model, the TBH will transition to a spheroidal BH once accretion processes have inflated the event horizon sufficiently or the angular momentum reduces. This provides a natural mechanism for the termination of quasar-like TBH activity within the universe and is amply supported by observations. The swelling of the toroidal event horizon due to mass capture generally overcomes the increasing angular momentum of the hole by the same process. Although the major radius of the torus may be increasing, the minor radius will tend to increase more rapidly leading to a topological transition. Immediately prior to the extinction of the TBH, the brightest and narrowest jets are anticipated to form, albeit with enhanced gravitational red-shifts as seen from infinity.

Fig.9 illustrates a number of features of the TBH quasar model and has been constructed using the inequality relations (12) to (15) from section 2. The shaded wedge represents the area within which TBHs can come into being directly from a SN implosion of a toroidal star. Here, it has been assumed that the seed star has constant density of 1400 kg m^{-3} and that 90% of the star's mass is ejected during the implosion ($\eta = 0.1$). TBH creation at radii R_1 below about $36 \times 10^9 \text{ m}$ is prohibited because the resulting R_2 would be much smaller than R_1 . Also, slightly beyond the left hand edge of the diagram at about $R_2/R_1 \sim 10^{-6}$ and beyond, collapse to neutron/electron degeneracy supported toroids becomes of concern and imposes additional restrictions so that the wedge shape does not continue indefinitely. Above the shaded wedge, it is impossible for the toroidal star to be sufficiently massive if, as it must be, $R_3 < R_1$ for the seed star and densities above 1400 kg m^{-3} are disallowed. Lightly shaded regions above and below the main wedge show how the diagram would be altered if different values for η were taken.

This diagram identifies the region at which quasar-like behaviour is to be expected from TBHs (progressively shaded vertical section) where R_2 is almost as large as R_1 and narrow jets are formed. The line defined by $R_2 = R_1$ is the quasar extinction boundary where the TBH becomes a spheroidal BH. Lines have been plotted to indicate contours of equal TBH mass and similarly for constant minor radius R_2 . Because the mass of the TBH will not diminish with time, constant mass boundaries can only be traversed in one

direction. There is a sufficiently broad birth zone spanning several orders of magnitude on each axis which enhances the probability that TBH creation is widespread at the centre of typical protogalaxies.

We would anticipate TBH birth to occur at the lower mass and lower R_2/R_1 ratio end of the shaded wedge because the seed stars required are very large even for these. One typical case has been presented on the diagram. For this example a toroidal star of mass $6 \times 10^6 M_\odot$ and radii $R_1 = 1.8 \times 10^{11} m$ and $R_3 = 8.7 \times 10^{10} m$ implodes after rapidly exhausting its fuel to leave a TBH of mass $6 \times 10^5 M_\odot$ and radii $R_1 = 1.8 \times 10^{11} m$ and $R_2 = 2.8 \times 10^6 m$. The accretion rate within this young galaxy is increasing so the TBH mass rapidly increases as does its angular momentum and angular velocity. The arrows of the evolutionary trace depict the evolutionary rate, fastest at the start then slowing down at higher masses such that the QSO behavioural phase can exist for a timescale several orders of magnitude greater than the formation time of the TBH. Somewhat inevitably, R_2 catches up with R_1 and the quasar phase is permanently discontinued, in this example when the mass reaches about $10^9 M_\odot$. For a given mass, the relationship between R_1 and R_2 will depend upon the angular velocity of the TBH which in turn is related to the angular accretion rate. In order to sustain a toroidal event horizon indefinitely, we would require an ever increasing rate of angular accretion. In practice, we would expect the accretion rate to be relatively low at the time of TBH birth, then rising swiftly before peaking and slowly decreasing thereafter. Accordingly, the jet formation phase is predicted to terminate due to the topological transition at the boundary where $R_2 \rightarrow R_1$.

8 DISCUSSION.

It has been qualitatively described how rotating TBHs can evolve from protogalactic gas clouds and accrete matter from the central galaxy until their inner apertures contract and highly focused relativistic jets are expected to result. It would be valuable to compare the jet divergence against red-shift measurements, as the model would imply that quasars within older galaxies would produce the most tightly focused jets, and that these are the immediate precursors of galaxies with relatively inactive nuclei. The direct observations of quasar objects in our universe suggests that TBHs are more than abstract constructs or mathematical curiosities. Classical general relativity still remains to be unified satisfactorily with quantum mechanics. Evidently, TBH stability is intertwined with this issue. If we are unable to experiment directly with intense spacetime curvature then we must be guided by astrophysical observations. The proposed stability of TBHs within our own universe allows us to metaphorically peel away the event horizon and study some aspects of the interior region. Such information is valuable in leading to a better understanding of quantum gravity and grand unification theories.

According to current solutions of general relativity for rotating TBHs, the cosmological constant in Einstein's equation (Λ) must be sufficiently negative in order to allow for the long term stability of such objects. Observational measurements of Λ all suggest that its value is approximately zero and are based upon universal expansion reliant on the behaviour of general relativity within weak gravitational fields. One possibility is that Λ is primarily a function of local spacetime curvature. Alternatively, it may be the presence of external matter (accretion disk and galaxy) which provides the necessary curvature and non-stationarity allowing long term TBH stability.

The TBH model holds the potential for us to understand the following characteristics of quasars: extreme jet energies, varying jet emergence angles, abrupt extinction, high gamma-ray radiation, the presence of heavy elements and the multiplicity of red-shifts in absorption spectra. None of these features are readily explicable using spheroidal black hole models. It is encouraging that the quasar mechanism as described here appears to lead naturally to plausible explanations for processes within SNe, microquasars and GRBs.

Numerical simulations are crucial if accurate comparisons with further detailed observations are to be made. Interferometric gravitational wave detectors being introduced in the next decade or two offer much promise in differentiating between BHs of different masses and spin rates. A TBH will be identifiable by its characteristic gravitational resonance and sensitive gravitational wave detectors could one day provide strong direct evidence of their existence.

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FIGURE CAPTIONS

- Figure 1. Evolution of gas distribution along galactic plane.
- Figure 2. Supernova implosion of toroidal star to form toroidal black hole.
- Figure 3. Cross-sectional view of toroidal black hole undergoing accretion.
- Figure 4. Relative clock rates in Schwarzschild gravity and nearby a moving point mass.
- Figure 5. Equatorial plane embedding diagram representing time dilation relative to distant observers.
- Figure 6. Transport of angular momentum from accretion disk to jets via TBH.
- Figure 7. Magnetic field lines surrounding conducting toroidal shell.
- Figure 8(a). Magnetic field strength external to an electrically conducting toroidal shell.
- Figure 8(b). Time dilation corrected magnetic field external to a conducting toroidal shell.
- Figure 9. Toroidal black hole creation, evolution and extinction.
- Figure 10. Internal pressure and fluid distribution within rotating spheres and cylinders.
- Figure 11. Pressure as a function of radius within rotating cylinder.

APPENDIX

Unlike the TBH structure, there are no known objections to the stability of rotating toroidal arrangements of neutron degenerate material in an asymptotically flat spacetime. It is reasonable to suppose that some form of stationary spacetime metric exists which describes the spacetime geometry external to the neutron ring itself. Butterworth & Ipser (1976) demonstrated that ergoregions can form when relativistic stars are rotating rapidly although absolute event horizons are absent. The significance of rotating toroidal neutron stars is that they adopt some of the characteristics of quasars. The quasar mechanism is potentially operating in a number of seemingly unrelated astrophysical phenomena.

Many stars, particularly the brighter Type Oe and Be, rotate very much more rapidly than our own Sun. Equatorial velocities in the range $300 \sim 700 \text{ km s}^{-1}$ are not uncommon as compared to 2 km s^{-1} for the Sun. It is thought that the majority of stars have high initial angular velocities but that coupling between solar winds and the inter-stellar medium cause a gradual decline in angular momentum. The brighter stars can be very short lived so we would anticipate them to retain high angular momentum once their fuel is exhausted. It is therefore worthwhile studying the internal structure of rapidly rotating stars undergoing gravitational collapse to determine the conditions where toroidal core configurations will arise.

In order to preserve analyticity, a simplified model is presented. Later it will be apparent that removal of any simplifications necessitates a numerical treatment. A uniformly rotating (constant angular velocity) ellipsoid would consist of ellipsoidal shells for which the assumption of uniform shell density is violated in the situations of interest so instead, uniformly rotating infinite cylinders are considered. A cross section is illustrated in Fig.10a) of the proposed structure of a rotating star composed of two immiscible, incompressible fluids with densities ρ_1 and ρ_2 where $\rho_2 > \rho_1$. The contours denote lines of equal hydrostatic pressure increasing from zero at the surface to a peak at the centre of the high density torus. Similarities with the distribution of matter in the Kerr BH are apparent but partially coincidental.

If the pressure and density within a cylinder composed of two fluids can be shown to reach a maximum at a distance away from the axis of rotation, Fig.10b), and that such a distribution is in equilibrium, then we may infer the validity of the arrangement in Fig.10a). A Newtonian analysis permits exact solutions by virtue of the superposition of cylindrical shells of differing densities, the linearity of (constant density) cylindrical gravity with radius as was established in section 2, the null gravity within infinite cylindrical shells and the gravitational equivalence of cylindrical shells to line masses in the exterior regions. We employ cylindrical coordinates r, ϕ, z and consider two immiscible and incompressible fluids of densities ρ_1 and ρ_2 with $\rho_2 > \rho_1$ rotating smoothly and uniformly. Temperature is neglected because we are particularly concerned with degenerate materials. The gravitational profile is not linear with radius because it is caused by three zones of different density: regions a), b) and c) with radii R_1, R_2 and R_3 respectively indicated in Fig.10b). When equilibrium is achieved, the resultant force on each fluid element is zero. The individual forces acting on the elements are due to the pressure gradient, the centripetal acceleration and the gravitational attraction which, by virtue of the cylindrical symmetry, need only be considered in the radial direction. It is straightforward to derive expressions for the derivative of pressure, P , with respect to radius for each of the three regions: -

Region a)

$$\frac{dP_a}{dr} = r\rho_1(\omega^2 - 2\pi G\rho_1) \quad (50)$$

Region b)

$$\frac{dP_b}{dr} = r\rho_2 \left\{ \omega^2 - 2\pi G \left[\rho_2 + \left(\frac{R_1^2}{r^2} \right) (\rho_1 - \rho_2) \right] \right\} \quad (51)$$

Region c)

$$\frac{dP_c}{dr} = r\rho_1 \left\{ \omega^2 - 2\pi G \left[\rho_2 + \left(\frac{R_1^2 - R_2^2}{r^2} \right) (\rho_1 - \rho_2) \right] \right\} \quad (52)$$

These expressions are readily integrated using the following boundary conditions: $P_c(R_3) = 0, P_b(R_2) = P_c(R_2)$ and $P_a(R_1) = P_b(R_1)$. It is immediately apparent from equation (50) that the pressure increases with radius providing a certain minimum angular velocity is exceeded: $\omega > \omega_{min} = \sqrt{2\pi G\rho_1}$. For physically meaningful results, the pressure must not become negative at radii occupied by matter. There are two circumstances where this might first arise: at the centre (when the cylinder becomes hollow) and at the surface (when surface fluid is shed due to centripetal forces in excess of gravitational forces). The latter

condition is simply expressed as $dP_c/dr > 0$ at $r \rightarrow R_3$ so we can also define a maximum angular velocity where $\omega_{max} > \omega_{min}$ and this is conveniently expressed as: -

$$\omega_{max} = \omega_{min} \times \sqrt{1 + \left(\frac{\rho_2 - \rho_1}{\rho_1} \right) \left(\frac{R_2^2 - R_1^2}{R_3^2} \right)} \quad (53)$$

In general, a wide range of angular velocities are available if the densities are very dissimilar whereas, as the densities of the two fluids approach one another, there is a much narrower range of values that ω can occupy above ω_{min} . A specific example is given in which the radii are in the ratio 1:2:3 for $R_1 : R_2 : R_3$ and the densities 1:2 for ρ_1 and ρ_2 . Results are plotted in Fig.11.

The diagram presents the pressure variation along the radius of a rotating infinite cylinder. Several curves have been plotted which correspond to different rates of rotation. For the example given, internal pressure remains positive up to $\omega \rightarrow 1.155\omega_{min}$. The stability of these results is trivial because the assumption of equilibrium was inherent in the model, all solutions are in neutral equilibrium including those at low angular velocity and the non-rotating case. The significance of ω_{min} is that stability cannot be achieved below this if the fluids become infinitesimally compressible because the density and pressure distributions would be qualitatively different. When a homogeneous rotating gravitating cylinder of (any) compressible fluid is considered, it transpires that stability of off-axis peak density arrangements is unattainable if the assumption of uniform rotation is used. This is evident upon inspection of equation (50) for cases where the pressure and density are positively correlated i.e. $dP/d\rho > 0$. This does not mean that an axial pressure-density peak will always result, the correct interpretation is that once the central angular velocity exceeds a certain value $\omega_c \sim \sqrt{2\pi G\rho_c}$ then axial density peaks are also unstable and differential rotation will occur (non-constant angular velocity). Furthermore, we can say that the angular velocity will be a generally decreasing function of radius for these systems, which coincides with the most physically realistic situation. Hence an off-axis density peak results which is akin to toroidal solutions of rotating gravitating spheroids. A numerical treatment must be employed for systems of this complexity, even though viscosity and temperature are here neglected.

When a rapidly rotating star undergoes core collapse, which processes are involved that contribute to the formation of a toroidal core? Primarily, because angular momentum is conserved, the angular velocity of the collapsed core increases significantly to a level in excess of ω_{min} and the angular velocity external to the core remains relatively constant. If the core is sufficiently dense, relativistic frame dragging contributes to the differential rotation. This radially decreasing rotation coupled to the fact that the pressure (and therefore the density) must increase with radius at the centre means that toroidal cores are to be expected within such rapidly rotating stars. The self-gravity of the torus then sustains a state of pseudo-equilibrium. For slowly rotating gravitating ellipsoids, the value of ω_{min} increases to $\sim \sqrt{4\pi G\rho_c}$. If the neutron core following gravitational collapse of our Sun has a density in excess of $\sim 3 \times 10^{18} \text{ kg m}^{-3}$ then it is conceivable that the core could become toroidal. If a more attainable collapse density of $10^{14} \text{ kg m}^{-3}$ were specified, then the Sun would need to rotate at a modest factor of 5.5 above its present rate for ω_{min} to be attained. The Sun's internal pressure would first vanish within the surface were it to rotate at a rate ~ 212 times above its present angular velocity, so the available rotation range for toroidal core collapse is generally broad for typical stars.

No known forms of matter are truly incompressible but it seems that a BH would expand under the influence of external pressure, behaving as though it were anti-compressible. This property somewhat resembles the negative heat capacity of BHs. Therefore we cannot say that a TBH embedded in a spinning star would be in equilibrium, but since the situation bears a strong resemblance to the described quasar mechanism, an intense explosion might be anticipated. Such a TBH could be of relatively low mass, perhaps below $0.1M_\odot$ as efficient jet generation processes begin to operate and halt its growth. There is therefore a definite possibility that the bulk of the remaining star is ejected and a low mass TBH/BH remnant is obtained. Prior to this, only primordial BHs were thought to constitute the BH population at masses below M_\odot . The Hawking evaporation of low mass BHs is only possible if the surface temperature exceeds that of the surrounding space which is known to be at least 3 degrees Kelvin and corresponds to a minuscule BH mass. Therefore these low mass BHs might contribute to dark matter but their evaporation is exceedingly unlikely.

Now let us consider the implications of toroidal matter distributions. In particular, toroidal neutron cored rotating white dwarves and isolated rotating neutron rings can exist. When the rotation rate is sufficiently high, such neutron rings can generate ergospheres, accumulate charge and produce intense magnetospheres providing negative energy states in much the same way as a quasar. We address the astronomical significance of neutron rings (or equivalently low mass TBHs) in the three circumstances mentioned previously.

Firstly, a binary system consisting of a neutron ring and, say, a nearby stellar companion could operate as follows: an accretion disk is formed around the neutron ring composed of material transported from the nearby star by gravitational and electromagnetic interactions. The central aperture of the neutron ring contains an ergosphere and an intense axial magnetic field due to a charge imbalance on the neutron ring. This then gives rise to anti-parallel jets aligned with the axis of rotation in a very similar manner to the quasar albeit on a much smaller scale. The mass of the neutron ring would be $\sim 6 - 9$ orders of magnitude below the mass of a typical quasar TBH. Microquasars have been observed within the confines of the Milky Way and are so called because they seem to obey simple scaling laws applied to quasars. Accretion of material from the companion star and jet formation will combine to decrease the overall angular momentum of these objects on a shorter timescale than that of the quasars, their unusual behaviour terminating when they transition to a spheroidal neutron star or BH after dissipating their angular momentum.

Relativistic galactic jet sources and their similarities with quasar outflows are reviewed by Mirabel and Rodriguez, 1999. From the limited microquasar observations described, it appears that the jet velocities have a bimodal distribution classified by $\nu_{jet} \approx 0.3c$ and $\nu_{jet} > 0.9c$. The likeliest explanation is that these are toroidal neutron rings and TBHs respectively. An interesting feature of some microquasars which is absent in quasars is their behaviour as the accretion disk is exhausted resulting in a sudden ejection of condensations (Mirabel et al, 1998). Existing steady state MHD models with continuous jets have difficulty accounting for this.

Secondly, the mechanism will operate in some of the more explosive Type II SNe. For example, a fuel starved, massive rotating star containing a toroidal arrangement of neutron degenerate matter embedded in a toroidal shell of electron degenerate matter within a spheroidal envelope of molecular density material could form during gravitational implosion. The internal neutron ring accumulates an overall charge that cannot be rapidly neutralised on a timescale comparable to that of the implosion. Again, a strong magnetic field arises in the central aperture of the neutron ring along with an ergoregion. The mechanism results in a ferocious outward explosion of matter from the centre of the SN in which a significant proportion of the star's mass is expelled anisotropically. Jets from SNe have been inferred from nearby hot-spots detected by optical speckle interferometry (Cen, 1999 and references therein). Evidence of highly anisotropic ejecta is provided by polarimetric SN observations (Wang et al, 1999). In an attempt to explain these phenomena, many papers have described numerical simulations of bipolar jet outflows from stellar cores without identifying the mechanism responsible for generating these jets. Usually a collapsar model is employed containing a spheroidal neutron or Kerr BH core whereby the absent jet generation mechanism is replaced by the crude assumption of rapid mass deposition onto the poles of the stellar core.

Thirdly, some gamma ray bursts (GRBs) could be generated by a mechanism similar to the microquasar. Consider a neutron ring embedded in a white dwarf star with a binary companion star providing a steady supply of material to the surrounding accretion disk. A periodic oscillation could be set up whereby the central neutron ring forms once sufficient matter has accumulated. This results in a brief and relatively anisotropic energy burst in which sufficient material is ejected to decrease the internal pressure sufficiently that the star becomes a pure white dwarf. Accretion of matter from the binary companion then continues, with the mass of the white dwarf slowly increasing until the process repeats. Recently, the burst GRB980425 was observed to coincide spatially and temporally with the SN 1998bw providing strong evidence for a common mechanism (Cen, 1999).

Some extragalactic observations of stellar orbital velocity profiles have suggested that Λ has a small and negative value, and that this negative Λ is equivalent to a constant energy density comparable to that of the presumed dark matter. Type Ia SNe at varying redshifts were used to determine Λ (Perlmutter et al, 1998) and concluded that universal expansion was accelerating. The set of briefest GRBs with ultra-relativistic jets may prove to be more reliable standard candles for future measurements.